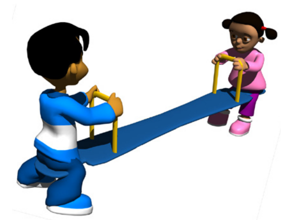
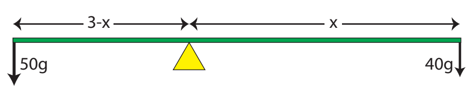
AP1 Rotation

1. A 50-kg boy and a 40-kg girl sit on opposite ends of a 3-meter see-saw. How far from the girl should the fulcrum be placed in order for the boy and girl to balance on opposite ends of the see-saw?



Answer: 1.67 m
In order for the children to balance, the net torque about the fulcrum must be zero, resulting in no angular acceleration. A diagram such as the one below may be helpful in developing the mathematical relationships.



$$\vec{\tau}_{net} = I\vec{\alpha} = 0 \rightarrow (50)(g)(3-x)-(40)(g)(x)=0 \rightarrow 500(3-x)-400x=0 \rightarrow 1500-500x-400x=0 \rightarrow$$
$$x = 1.67m$$

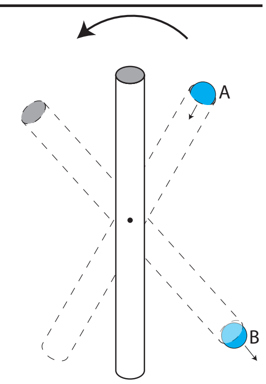
EK: 3.F.2 The presence of a net torque along any axis will cause a rigid system to change its rotational motion or an object to change its rotational motion about that axis.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 3.F.2.1 The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis.

AP1 Rotation

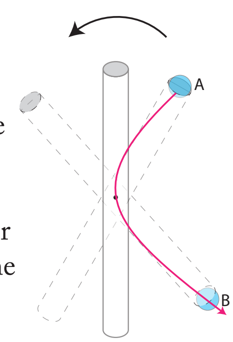
2. A uniform hollow tube of length L rotates vertically about its center of mass as shown. A ball is dropped into the tube at position A, and exits a short time later at position B. From the perspective of a stationary observer watching the tube rotate, the distance the ball travels is



(A) less than L
(B) greater than L
(C) equal to L

Answer: (B) greater than L

Though the displacement of the ball at B from its initial position at A is less than the length of the rod, L, the distance the ball travels is greater than L from the point of view of an external observer watching the tube rotate. Imagine the tube is transparent as you observe the path of the ball from a stationary reference point. You would observe the ball traveling a curved path from A to B, as shown at right. The length of A to the center point is one radius, and since the ball takes a curved path, it travels a distance greater than that radius. The same occurs from the center point to point B. Therefore, the ball travels a distance greater than the length of the tube from the perspective of an external observer at a stationary reverence point.

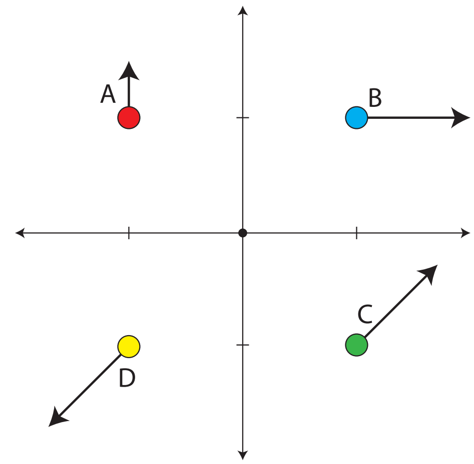


EK: 3.A.1.c An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration. A choice of reference frame determines the direction and the magnitude of each of these quantities.

SP: 1.3 The student can refine representations and models of natural or man–made phenomena and systems in the domain. 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively.

LO: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations.

3. Four particles, each of mass *M*, move in the x-y plane with varying velocities as shown in the diagram. The velocity vectors are drawn to scale. Rank the magnitude of the angular momentum about the origin for each particle from largest to smallest.



Answer: C, B, A, D
C has the highest angular momentum as it has the maximum velocity and the angle between the position vector to the mass and the velocity vector is 90 degrees. B has the next highest angular momentum as it has the maximum velocity, and the angle between the position vector to the mass and the velocity vector is 45 degrees. A has the third highest angular momentum as it has half the maximum velocity, and the angle between the position vector to the mass and the velocity vector is 45 degrees. D has the lowest angular momentum (0) about the origin since the position vector to the mass and its velocity are parallel.

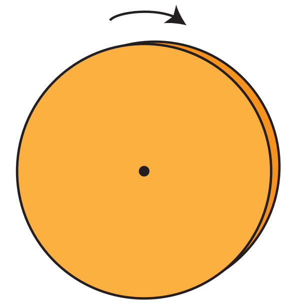
EK: 5.E.2 The angular momentum of a system is determined by the locations and velocities of the objects that make up the system. The rotational inertia of an object or system depends upon the distribution of mass within the object or system. Changes in the radius of a system or in the distribution of mass within the system result in changes in the system's rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.3 The student can estimate numerically quantities that describe natural phenomena.

LO: 5.E.2.1 The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses.

AP1 Rotation

4. A disc rotates clockwise about its axis as shown in the diagram. The direction of the angular momentum vector is:



(A) out of the plane of the page
(B) into the plane of the page
(C) toward the top of the page
(D) toward the bottom of the page

Answer: (B) into the plane of the page
The direction of the angular momentum vector is given by the right-hand-rule. In this case, wrap the fingers of your right-hand around the axle in the direction of the rotational velocity, and your thumb points in the direction of the angular momentum vector (into the plane of the page.)

EK: 4.D.1 Torque, angular velocity, angular acceleration, and angular momentum are vectors and can be characterized as positive or negative depending upon whether they give rise to or correspond to counterclockwise or clockwise rotation with respect to an axis.

SP: 1.2 The student can describe representations and models of natural or man–made phenomena and systems in the domain.

LO: 4.D.1.1 The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system.

5. A hoop with moment of inertia $I=0.1\ kg \cdot m^2$ spins about a frictionless axle with an angular velocity of 5 radians per second. At what radius from the center of the hoop should a force of 2 newtons be applied for 3 seconds in order to accelerate the hoop to an angular speed of 10 radians per second?

(A) 8.3 cm
(B) 12.5 cm
(C) 16.7 cm
(D) 25 cm

Answer: (A) 8.3 cm

A net torque will change the angular momentum of the hoop. Solving for the distance at which the force must be applied to create the appropriate torque:

$$\tau_{net} = I\alpha \rightarrow Fr = I\left(\frac{\omega_f - \omega_i}{t}\right) \rightarrow r = \frac{I}{F}\left(\frac{\omega_f - \omega_i}{t}\right) = \frac{0.1}{2}\left(\frac{10-5}{3}\right) = 0.083m = 8.3cm$$

EK: 3.F.2 The presence of a net torque along any axis will cause a rigid system to change its rotational motion or an object to change its rotational motion about that axis. 3.F.3 A torque exerted on an object can change the angular momentum of an object.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 3.F.2.1 The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. 3.F.3.2 In an unfamiliar context or using representations beyond equations, the student is able to justify the selection of a mathematical routine to solve for the change in angular momentum of an object caused by torques exerted on the object.

6. Jean stands at the exact center of a large spinning frictionless uniform disk of mass M and radius R with moment of inertia $I=\frac{1}{2}MR^2$. As she walks from the center to the edge of the disk, the angular speed of the disk is quartered. Which of the following statements is true?

(A) Jean's mass is less than the mass of the disk.
(B) Jean's mass is equal to the mass of the disk.
(C) Jean's mass is between the mass of the disk and twice the mass of the disk.
(D) Jean's mass is more than twice the mass of the disk.

Answer: (C) Jean's mass is between the mass of the disk and twice the mass of the disk.

The initial moment of inertia of the system is approximately $I=\frac{1}{2}MR^2$. Jean's mass does not contribute significantly to this moment of inertia as she stands at the exact center of the large disk. As she walks from the center of the disk to the edge of the disk, however, she adds her moment of inertia to that of the disk, so that the total moment of inertia of the system is now $I= \frac{1}{2}MR^2 + mR^2$. Calling the initial angular speed of the disk ω and applying the law of conservation of angular momentum, find Jean's mass (m) as follows:

$$L_i = L_f \rightarrow \tfrac{1}{2}MR^2\omega = (\tfrac{1}{2}MR^2 + mR^2)\frac{\omega}{4} \rightarrow 4M = M + 2m \rightarrow m = \tfrac{3}{2}M$$

EK: 5.E.1 If the net external torque exerted on the system is zero, the angular momentum of the system does not change. 5.E.2 The angular momentum of a system is determined by the locations and velocities of the objects that make up the system. The rotational inertia of an object or system depends upon the distribution of mass within the object or system. Changes in the radius of a system or in the distribution of mass within the system result in changes in the system's rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 3.3 The student can evaluate scientific questions.

LO: 5.E.1.2 The student is able to make calculations of quantities related to the angular momentum of a system when the net external torque on the system is zero. 5.E.2.1 The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses.

7. A spinning plate in a microwave with moment of inertia I rotates about its center of mass at a constant angular speed ω. When the microwave ends its cook cycle, the plate comes to rest in time Δt due to a constant frictional force F applied a distance r from the axis of rotation. What is the magnitude of the frictional force F?

(A) $F = \frac{I\omega}{r\Delta t}$

(B) $F = \frac{Ir}{\omega\Delta t}$

(C) $F = \frac{\omega r^2}{I\Delta t}$

(D) $F = \frac{Ir^2}{\omega\Delta t}$

Answer: (A) $F = \frac{I\omega}{r\Delta t}$

The change in angular momentum of the plate is given by the product of the net torque and the time interval over which it is applied. Therefore:

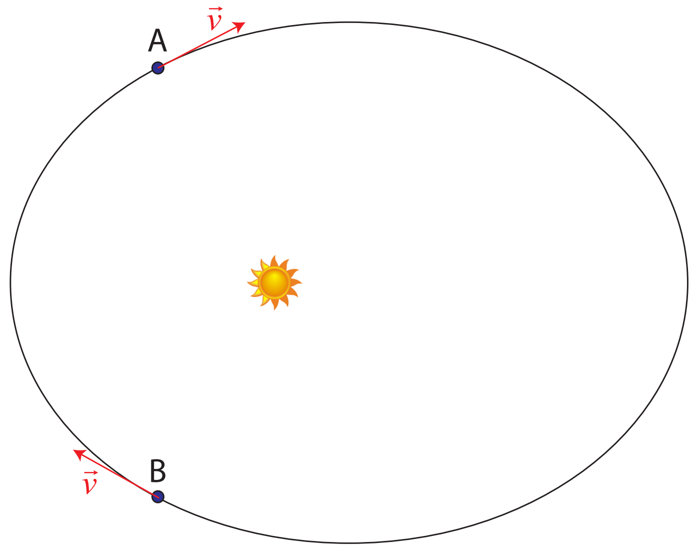
$$\Delta L = I\Delta\omega = \tau\Delta t \rightarrow I\omega = Fr\Delta t \rightarrow F = \frac{I\omega}{r\Delta t}$$

EK: 3.F.2 The presence of a net torque along any axis will cause a rigid system to change its rotational motion or an object to change its rotational motion about that axis. 3.F.3 A torque exerted on an object can change the angular momentum of an object. 4.D.3 The change in angular momentum is given by the product of the average torque and the time interval during which the torque is exerted.

SP: 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 3.F.2.1 The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. 3.F.3.2 In an unfamiliar context or using representations beyond equations, the student is able to justify the selection of a mathematical routine to solve for the change in angular momentum of an object caused by torques exerted on the object. 4.D.3.1 The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum.

8. A planet orbits a sun in an elliptical orbit as shown. Which principles of physics most clearly and directly explain why the speed of the planet is the same at positions A and B? Select two answers.



(A) Conservation of Energy
(B) Conservation of Angular Velocity
(C) Conservation of Angular Momentum
(D) Conservation of Charge

Answer: (A) Conservation of Energy and (C) Conservation of Angular Momentum

(A) Ultimately conservation of energy can lead you to this conclusion, though there are several steps analyzing kinetic and gravitational potential energy to get there.

(B) There is no such law as Conservation of Angular Velocity. This is silly.

(C) An analysis using conservation of Angular Momentum leads directly to $m_A v_A R_A \sin\theta_A = m_B v_B R_B \sin\theta_B$, and given that R, θ, and m are the same at positions A and B, you have a clear and direct path to $v_A = v_B$.

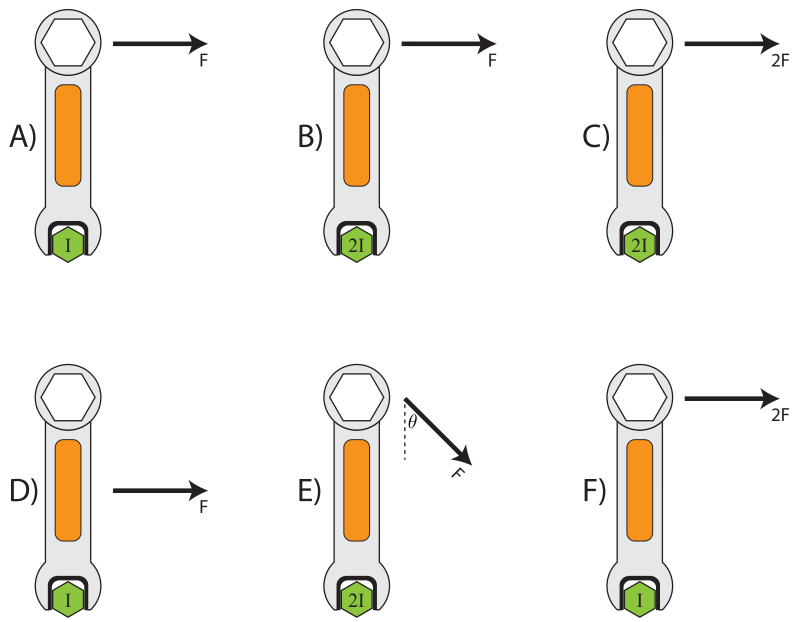
(D) Conservation of electrical charge, though true, does not help you with this problem.

EK: 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings. 5.E.2 The angular momentum of a system is determined by the locations and velocities of the objects that make up the system. The rotational inertia of an object or system depends upon the distribution of mass within the object or system. Changes in the radius of a system or in the distribution of mass within the system result in changes in the system's rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum.

SP: 6.1 The student can justify claims with evidence. 6.5 The student can evaluate alternative scientific explanations.

LO: 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. 5.E.2.1 The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses.

9. A given force is applied to a wrench to turn a bolt of specific rotational inertia I which rotates freely about its center as shown in the following diagrams. Which of the following correctly ranks the resulting angular acceleration of the bolt?



(A) $\alpha_F > \alpha_A = \alpha_C > \alpha_B = \alpha_D > \alpha_E$

(B) $\alpha_C = \alpha_F > \alpha_A = \alpha_B > \alpha_D > \alpha_E$

(C) $\alpha_A = \alpha_F > \alpha_C = \alpha_E > \alpha_B = \alpha_D$

(D) $\alpha_D = \alpha_F > \alpha_A = \alpha_C > \alpha_B > \alpha_E$

Answer: (A) F > A=C > B=D > E

Net torque is equal to the product of the angular acceleration and the rotational inertia, therefore angular acceleration is net torque divided by moment of inertia. Assume the length of the entire wrench is L.

Diagram	Torque	Rotational I	Angular Acceleration
A	FL	I	FL/I
B	FL	2I	FL/2I
C	2FL	2I	FL/I
D	FL/2	I	FL/2I
E	FLsin45°	2I	FL√2/4I
F	2FL	I	2FL/I

EK: 3.F.1 Only the force component perpendicular to the line connecting the axis of rotation and the point of application of the force results in a torque about that axis.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 3.F.1.2 The student is able to compare the torques on an object caused by various forces. 3.F.1.3 The student is able to estimate the torque on an object caused by various forces in comparison to other situations.

AP1 Rotation

10. Gina races her bike across a horizontal path. Suddenly, a squirrel runs in front of her. Gina slams on both her front and rear brakes, which results in the bike flipping over the front wheel and Gina flying over the handle bars. Which of the following do NOT contribute to an explanation of why the bike flips and Gina flies over the handlebars?

(A) Gina has a tendency to continue moving at a constant velocity, so while the bike stops, Gina continues her previous motion.
(B) Conservation of angular momentum of the bike and wheels indicates that if the wheels stop spinning in one direction, the bike must spin in the opposite direction.
(C) The large negative acceleration of the bike/rider system reduces the moment of inertia of the system, increasing the system's angular acceleration and causing a rotation of the bike.
(D) The force of the applied brakes at a distance from the center of mass of the bike and rider produces a net torque on the bike, causing a rotation bringing the back wheel of the back up.

Answer: (C) The large negative acceleration of the bike/rider system reduces the moment of inertia of the system, increasing the system's angular acceleration and causing a rotation of the bike.

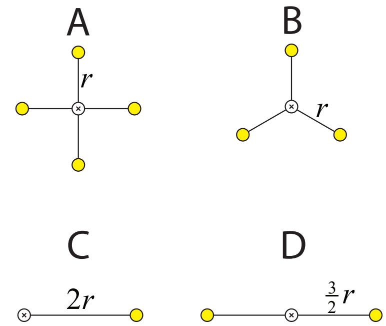
(A) is true, as a restatement of Newton's 1st Law of Motion. (B) is true, though the effect may be rather small if the mass of the wheels is relatively small compared to the mass of the rest of the bike. (D) is true as the force of friction provides a net torque on the bike (similar to how a motorcycle may "pop a wheelie" when accelerated quickly). (C) is completely made up, however, and is incorrect.

EK: 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces. 3.F.2 The presence of a net torque along any axis will cause a rigid system to change its rotational motion or an object to change its rotational motion about that axis. 3.F.3 A torque exerted on an object can change the angular momentum of an object.

SP: 6.2 The student can construct explanations of phenomena based on evidence produced through scientific practices. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 3.B.1.1 The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. 3.F.2.1 The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. 3.F.3.1 The student is able to predict the behavior of rotational collision situations by the same processes that are used to analyze linear collision situations using an analogy between impulse and change of linear momentum and angular impulse and change of angular momentum.

11. Identical point masses are arranged in space and connected by massless rods in four different configurations, as shown in the diagram below. Rank the moment of inertia of the configurations from greatest to least, assuming the masses are rotated about the point indicated with an x.



Answer: D>A=C>B

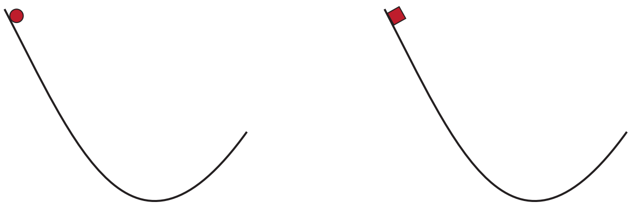
Moment of inertia can be obtained by taking the sum of the masses times the square of their distance from the axis of rotation.

EK: 5.E.2 The angular momentum of a system is determined by the locations and velocities of the objects that make up the system. The rotational inertia of an object or system depends upon the distribution of mass within the object or system. Changes in the radius of a system or in the distribution of mass within the system result in changes in the system's rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 5.E.2.1 The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses.

12. A ball and a block of equal mass are situated on ramps with the same shape. The objects are released from the same height. The ball rolls without slipping, and the block travels without friction. After leaving the ramp, which object travels higher and why?



(A) The ball travels higher because it leaves the ramp with a higher speed.
(B) The ball travels higher because it gains rotational kinetic energy alongs its path.
(C) The block travels higher because it experiences no rotation.
(D) The block travels higher because energy is not conserved in a rolling system.

Answer: (C) The block travels higher because it experiences no rotation.

The ball spins as it rolls down the ramp, converting its gravitational potential energy to rotational kinetic energy and translational kinetic energy. The block, on the other hand, doesn't spin, therefore all of its gravitational potential energy is converted into translational kinetic energy. The block therefore leaves the ramp with more speed at the same angle as the ball, therefore it travels higher.

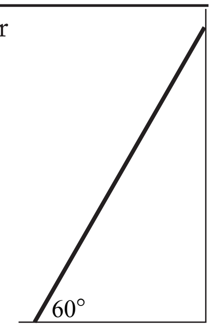
EK: 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy. 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models. 6.5 The student can evaluate alternative scientific explanations. 7.1 The student can connect phenomena and models across spatial and temporal scales.

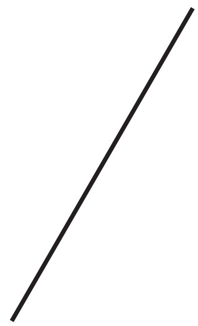
LO: 4.C.1.2 The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations.

AP1 Rotation

13. A 20-kg ladder of length 8 m sits against a frictionless wall at an angle of 60 degrees. The ladder just barely keeps from slipping.



(a) On the diagram below, draw and label the forces acting on the ladder.

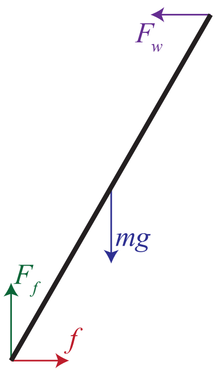


(b) Determine the force of friction of the floor on the ladder.

(c) Determine the coefficient of friction between the ladder and the floor.

Answer:

(a)



(b) Utilize Newton's 2nd Law in the x- and y-directions, as well as Newton's 2nd Law for Rotation, to determine the force of friction *f*.

$$F_{net_X} = f - F_w = 0 \rightarrow f = F_w$$

$$F_{net_Y} = F_f - mg = 0 \rightarrow F_f = mg = (20kg)(10 \,{}^{m}\!/\!_{s^2}) = 200N$$

$$\tau_{net} = -4mg\cos 60° + 8F_w \sin 60° \rightarrow F_w = \frac{4mg\cos 60°}{8\sin 60°} = 57.7N$$

(c) Solve for the coefficient of friction.

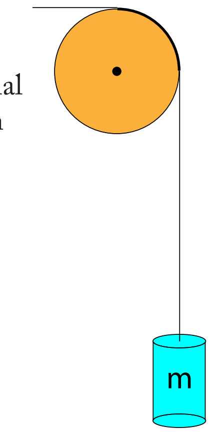
$$f = \mu N \rightarrow \mu = \frac{f}{N} = \frac{57.7N}{200N} = 0.29$$

EK: 3.A.2 Forces are described by vectors. 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces. 3.F.1 Only the force component perpendicular to the line connecting the axis of rotation and the point of application of the force results in a torque about that axis.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 3.A.2.1 The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. 3.B.1.3 The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. 3.F.1.5 The student is able to calculate torques on a two-dimensional system in static equilibrium, by examining a representation or model (such as a diagram or physical construction).

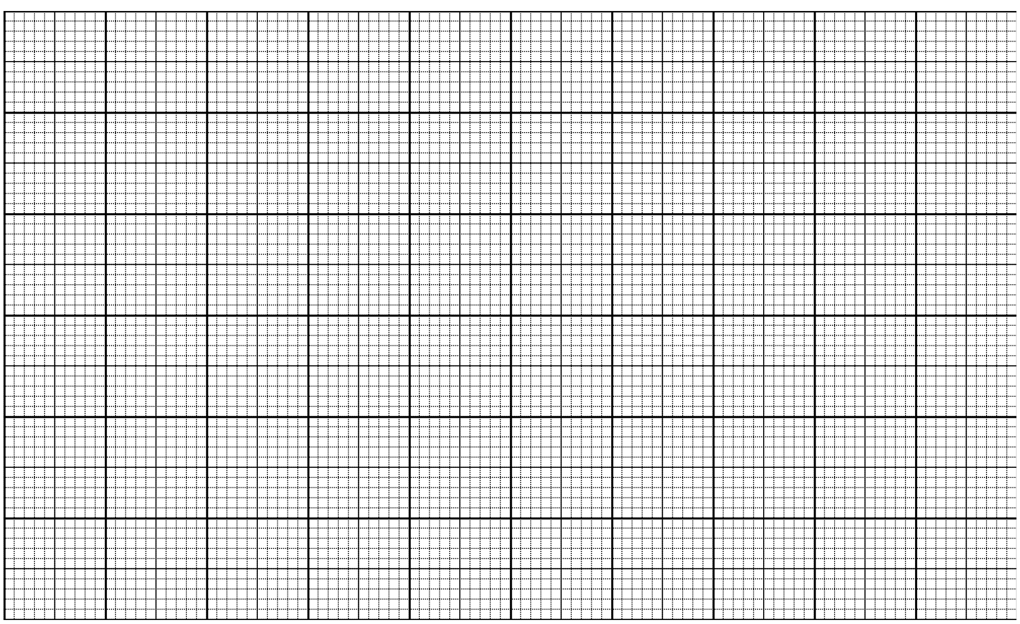
14. A student designs an experiment in which a mass (m) attached to a one-meter-long string is wrapped around a pulley of mass M=1 kg and radius R=0.1 m and dropped. The pulley includes a sensor which measures and records its rotational velocity. The mass is dropped from rest and the final rotational velocity of the pulley as well as the time the mass is in the air is measured. Data is shown in the table below.



m (kg)	ω_f (rad/s)	time (s)	α (rad/s^2)
0.2	24	0.84	
0.4	30	0.68	
0.6	33	0.61	
0.8	35	0.58	
1.0	36	0.55	

(a) Complete the table above by calculating the angular acceleration for each of the hanging masses.

(b) Plot the angular acceleration vs. the hanging mass. Make sure to label all axes appropriately.



(c) Using your plot, estimate the angular acceleration of the pulley for a hanging mass of 500g.

(d) Assume the sensor which measures the rotational velocity of the pulley breaks. Explain how you could accomplish this experiment without using the sensor.

(e) Estimate the time it will take for a 500g mass attached to the string to fall all the way through the pulley. Show all work.

EK: 3.F.1 Only the force component perpendicular to the line connecting the axis of rotation and the point of application of the force results in a torque about that axis. 4.D.1 Torque, angular velocity, angular acceleration, and angular momentum are vectors and can be characterized as positive or negative depending upon whether they give rise to or correspond to counterclockwise or clockwise rotation with respect to an axis.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

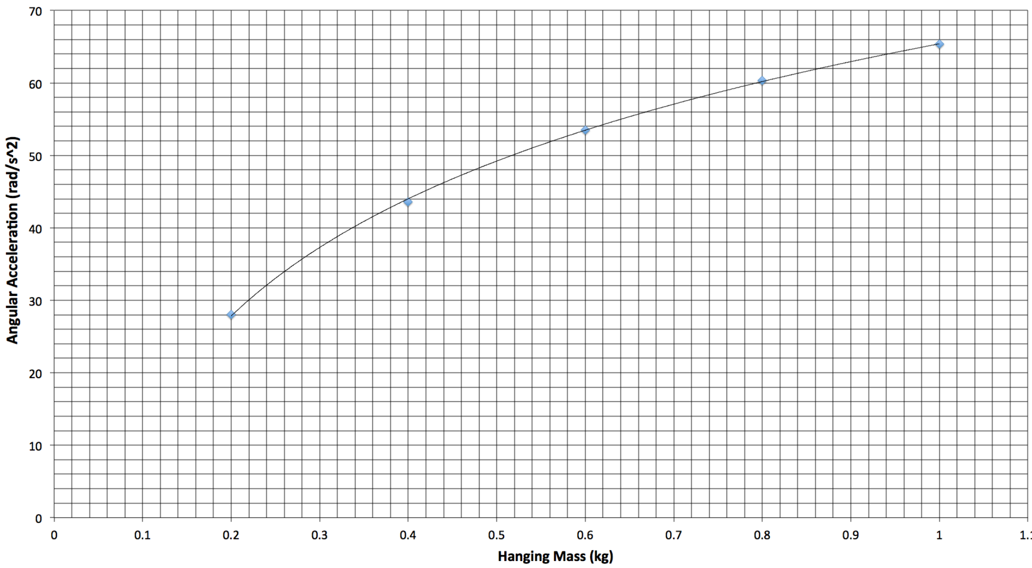
LO: 3.F.1.4 The student is able to design an experiment and analyze data testing a question about torques in a balanced rigid system. 4.D.1.1 The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. 4.D.1.2 The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, and angular momentum can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data.

Answer:

(a)

m (kg)	ω_f (rad/s)	time (s)	α (rad/s^2)
0.2	24	0.84	28
0.4	30	0.68	44
0.6	33	0.61	54
0.8	35	0.58	60
1.0	36	0.55	65

(b)



(c) $\alpha \approx 49\, {}^{rad}\!/_{s^2}$

(d) Knowing the time it takes for the hanging mass to fall one meter, you could calculate the linear acceleration of the falling mass, which would correspond to the angular acceleration of the pulley using the relationship α=a/R.

(e) The linear acceleration can be determined from a simple transformation:

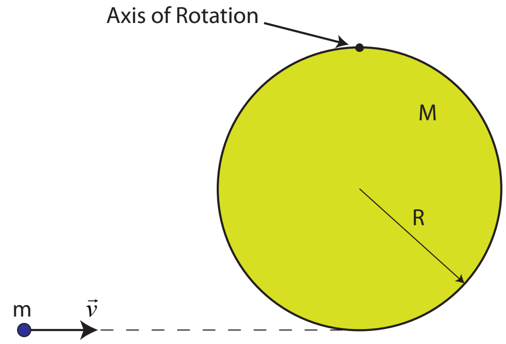
$$a = \alpha R = (49\, {}^{rad}\!/_{s^2})(0.1m) = 4.9\, {}^{m}\!/_{s^2}$$

Then, time can be determined using basic kinematics:

$$\Delta y = v_0 t + \tfrac{1}{2} a_y t^2 \xrightarrow{v_0=0} t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(1m)}{4.9\, {}^{m}\!/_{s^2}}} = 0.64s$$

15. A particle of mass m is launched with velocity v toward a uniform disk of mass M and radius R which can rotate about a point on its edge as shown. The disk is initially at rest. After the particle strikes the edge of the disk and sticks, the magnitude of the final angular velocity of the disk-particle system is given by:

$$|\vec{\omega}| = \left(\frac{4m}{8m+3M}\right)\left(\frac{v}{R}\right)$$



Determine the moment of inertia of the disk about its axis of rotation.

Answer: $I_{disk} = \frac{3}{2}MR^2$

Treating the disk and particle as a single system, the total angular momentum of the system must be conserved in the absence of external torques, therefore the initial angular momentum of the system must equal the final angular momentum of the system.

$$L_i = L_f = I\omega_f$$

Substituting in the initial angular momentum of the particle and the final angular velocity of the system and solving for the total moment of inertia of the disk-particle system:

$$mv(2R) = I_{total}\omega_f = I_{total}\left(\frac{4m}{8m+3M}\right)\left(\frac{v}{R}\right) \to I_{total} = 4mR^2 + \frac{3}{2}MR^2$$

Finally, recognizing the total moment of inertia of the system is the sum of the moment of inertia of the particle and the moment of inertia of the disk, solve for the moment of inertia of the disk.

$$I_{disk} = I_{total} - I_{particle} = (4mR^2 + \frac{3}{2}MR^2) - (m(2R)^2) = \frac{3}{2}MR^2 \to$$

$$I_{disk} = \frac{3}{2}MR^2$$

(Note that if students are familiar with the Parallel Axis Theorem this can be determined much more simply, though the PAT is not formally within the scope of the AP Physics 1 curriculum.)

EK: 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings. 5.E.2 The angular momentum of a system is determined by the locations and velocities of the objects that make up the system. The rotational inertia of an object or system depends upon the distribution of mass within the object or system. Changes in the radius of a system or in the distribution of mass within the system result in changes in the system's rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. 5.E.2.1 The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses.

AP1 Rotation

16. A round object of mass m and radius r sits at the top of a track of length L inclined at an angle of θ with the horizontal.

(a) Derive an expression for the gravitational potential energy of the object in terms of m, L, and θ.

A student wishes to experimentally determine the object's moment of inertia, I, by adjusting the angle of the ramp and observing the behavior of the object as it rolls down the ramp without slipping.

(b) Describe an experimental procedure the student could use to collect the data necessary, including any equipment the student would need. Your description should include a listing of the independent and dependent variable(s).

(c) Derive an expression for the velocity of the object at the bottom of the ramp in terms of the length of the ramp, L, and the time it takes to travel down the ramp, t.

(d) Derive an expression for the moment of inertia of the object in terms of m, r, θ, t, L, and any fundamental constants.

(e) The student plots the square of the time it takes for the object to travel down the ramp, t^2, as a function of $1/\sin(\theta)$ to obtain a linear graph. How should the student determine the moment of inertia of the object from this graph? Highlight the calculation(s) used.

(f) Now suppose that you were not given the radius of the object. Describe an experimental procedure you could use to determine it, including any equipment you would need.

(g) Derive the expression for the moment of inertia of the object in terms of m, r, θ, t, L, and any fundamental constants using an alternate approach to your method from part (d).

EK: 3.B.1 If an object of interest interacts with several other objects, the net force is the vector sum of the individual forces. 3.F.1 Only the force component perpendicular to the line connecting the axis of rotation and the point of application of the force results in a torque about that axis. 3.F.2 The presence of a net torque along any axis will cause a rigid system to change its rotational motion or an object to change its rotational motion about that axis. 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy.

SP: 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 4.1 The student can justify the selection of the kind of data needed to answer a particular scientific question. 4.2 The student can design a plan for collecting data to answer a particular scientific question. 5.2 The student can refine observations and measurements based on data analysis. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 3.B.1.2 The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces. 3.F.1.1 The student is able to use representations of the relationship between force and torque. 3.F.2.2 The student is able to plan data collection and analysis strategies designed to test the relationship between a torque exerted on an object and the change in angular velocity of that object about an axis. 4.C.1.2 The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system.

Answer:

(a) $U_g = MgL\sin\theta$

(b) Using a stopwatch, the student could start the stopwatch when the object is released from rest, and halt the stopwatch when the object reaches the bottom of the incline, repeating this for varying levels of inclination and recording the independent variable (θ) and the dependent variable (t) in a data table. Other equivalent answers are acceptable, such as utilizing photogates to determine the time, or determining the speed of the object as it reaches the bottom of the incline.

(c) $v = \frac{2L}{t}$

(d) $$mgL\sin\theta = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \rightarrow 2mgL\sin\theta = mv^2 + I\omega^2 \rightarrow I = \frac{2mgL\sin\theta}{\omega^2} - \frac{mv^2}{\omega^2} \xrightarrow[\omega = v/r]{v=\omega r}$$

$$I = \frac{2mgL\sin\theta r^2}{v^2} - mr^2 = mr^2\left(\frac{2gL\sin\theta}{v^2} - 1\right) \xrightarrow{v^2 = 4L^2/t^2} I = mr^2\left(\frac{2gLt^2\sin\theta}{4L^2} - 1\right) \rightarrow$$

$$I = mr^2\left(\frac{gt^2\sin\theta}{2L} - 1\right)$$

(e) Plotting the function $t^2 = \left[\frac{2L(I+mr^2)}{mr^2 g}\right]\left(\frac{1}{\sin\theta}\right)$ indicates that the slope of the line will give you the bracketed expression. Therefore, to find the moment of inertia, find the slope of the line first, then you may rearrange this equation to find $I = mr^2\left(\frac{g \times slope}{2L} - 1\right)$

(f) One possible answer includes wrapping a string around the circumference of the object, marking off the circumference, and measuring its length. The radius, then, is the circumference divided by 2π.

(g) The moment of inertia may be determined by a conservation of energy approach (per guidance above), as well as from a Newton's 2nd Law approach:

$$F_{net_x} = mg\sin\theta - f = ma$$

$$\tau_{net} = fr = I\alpha \rightarrow f = \frac{I\alpha}{r}$$

$$mg\sin\theta - \frac{I\alpha}{r} = ma \xrightarrow{\alpha = a/r} mg\sin\theta - \frac{Ia}{r^2} = ma \rightarrow \frac{Ia}{r^2} = mg\sin\theta - ma \rightarrow$$

$$I = mr^2\left(\frac{g\sin\theta}{a} - 1\right) \xrightarrow{a = 2L/t^2} I = mr^2\left(\frac{gt^2\sin\theta}{2L} - 1\right)$$